Practice Midterm 1

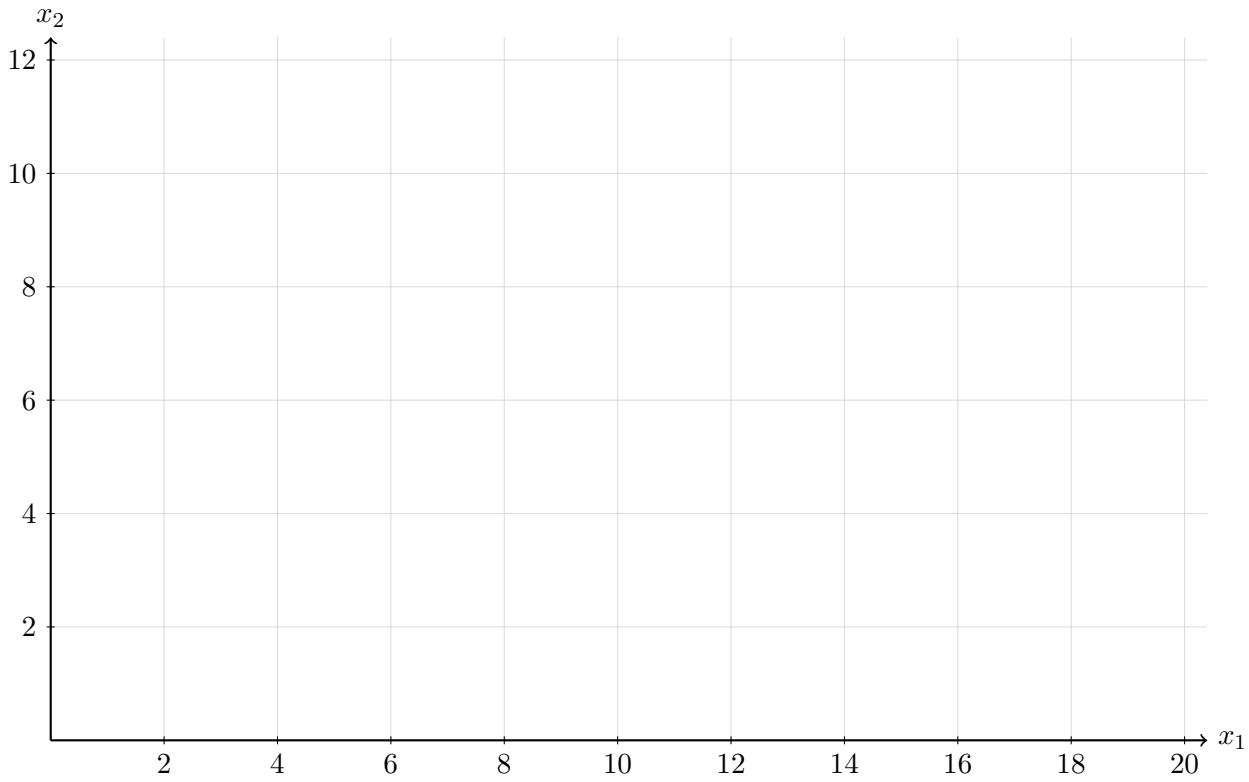
Problem 1. Will has the following utility function: $u(x_1, x_2) = \min\{x_1, 2x_2\}$

(a) Graph three of his indifference curves such that his utility equals 2, 5 and 10 utils. Label them IC_1, IC_2, and IC_3 respectively.

(b) If the price of x_1 is \$2, x_2 is \$4, and Will has an income of \$20 what is his budget line? Graph it below and label the line BL_1. What is Will's optimal bundle?

(c) If the price of x_1 is \$2, x_2 is \$4, and Will instead has an income of \$40 what is his budget line? Graph it below and the new line BL_2. What is Will's new optimal bundle?

(d) If the price of x_1 is \$8, x_2 is \$4, and Will has an income of \$20 what is his budget line? Graph it below and label the line BL_3. What is Will's optimal bundle?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Problem 2. Jon likes to spend his time ice sculpting and theorizing. His preferences over these two activities are represented by the utility function $u(x_t, x_i) = 2x_i + 50x_t - 2x_t^2$. His demand for theorizing is represented by the function $x_t(p_t, p_i) = 12.5 - \frac{p_t}{2p_i}$

(a) What is Jon's inverse demand of theorizing?

(b) If Jon has 200 hours of free time to spend and each ice sculpting takes 2 hours while theorizing takes 4 hours, what is his budget line? What is his optimal bundle?

(c) Suppose instead that it takes Jon 2 hours to theorize, what is his new optimal bundle?

Problem 3. Raoul likes to consume fancy Italian espresso and coffee. His utility function over the two goods is $u(e, c) = 6e + 3c$. He has a budget of \$60 to spend on the two goods where the price of espresso is \$10 and coffee is also \$10.

(a) Write down and graph his budget line below. Label it BL_1

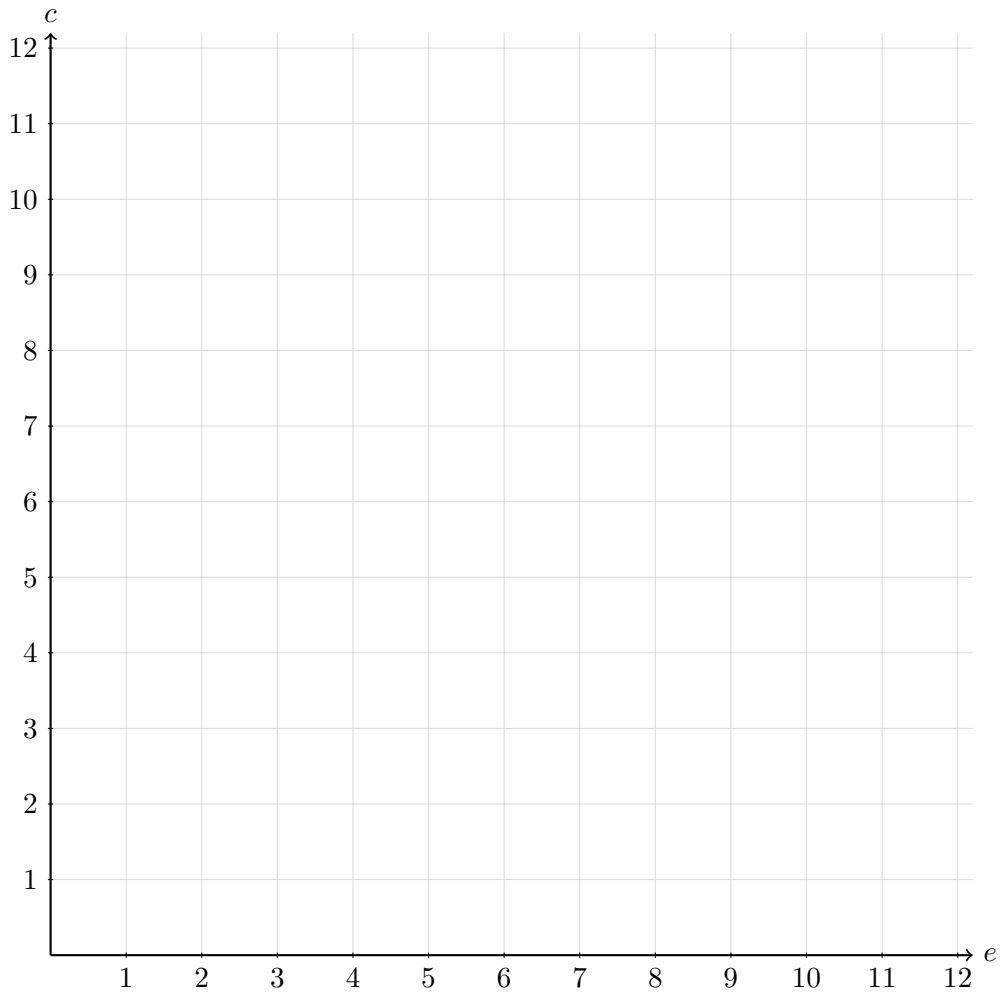
(b) Graph 3 of his indifference curves such that they represent a utility of 12, 24, and 36. Label them IC_1, IC_2, and IC_3 respectively.

(c) What kind of preference do these indifference curves represent?

(d) What is Raoul's optimal bundle?

(e) Suppose the government issues a subsidy of \$5 for each bag of coffee beans bought from Italy. What is Raoul's new budget line. Graph it below and label it BL_2.

(f) What is his new optimal bundle with the subsidy?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Problem 4. Jeff is a big animal lover. Specifically, he likes to adopt cats (c) and bunnies (b). His preference over the two pets is represent by the utility function of $u(c, b) = 3c^2 + 2b$. Let m be his income, p_c the adoption price of cats, and p_b the adoption price of bunnies.

(a) If the current adoption prices are \$60 for cats and \$10 for bunnies and Jeff has an income of \$220 what is his budget line?

(b) If Jeff's demand function for cats is $c(p_c, p_b) = \frac{p_c}{3p_b}$, how many of each animal will he adopt?

(c) If the adoption price of cats fell to \$40, what would Jeff's income need to be for him to still be able to afford his original optimal bundle? Denote this new income m'

(d) Under the modified income and changed price, how many of each animal would Jeff adopt?

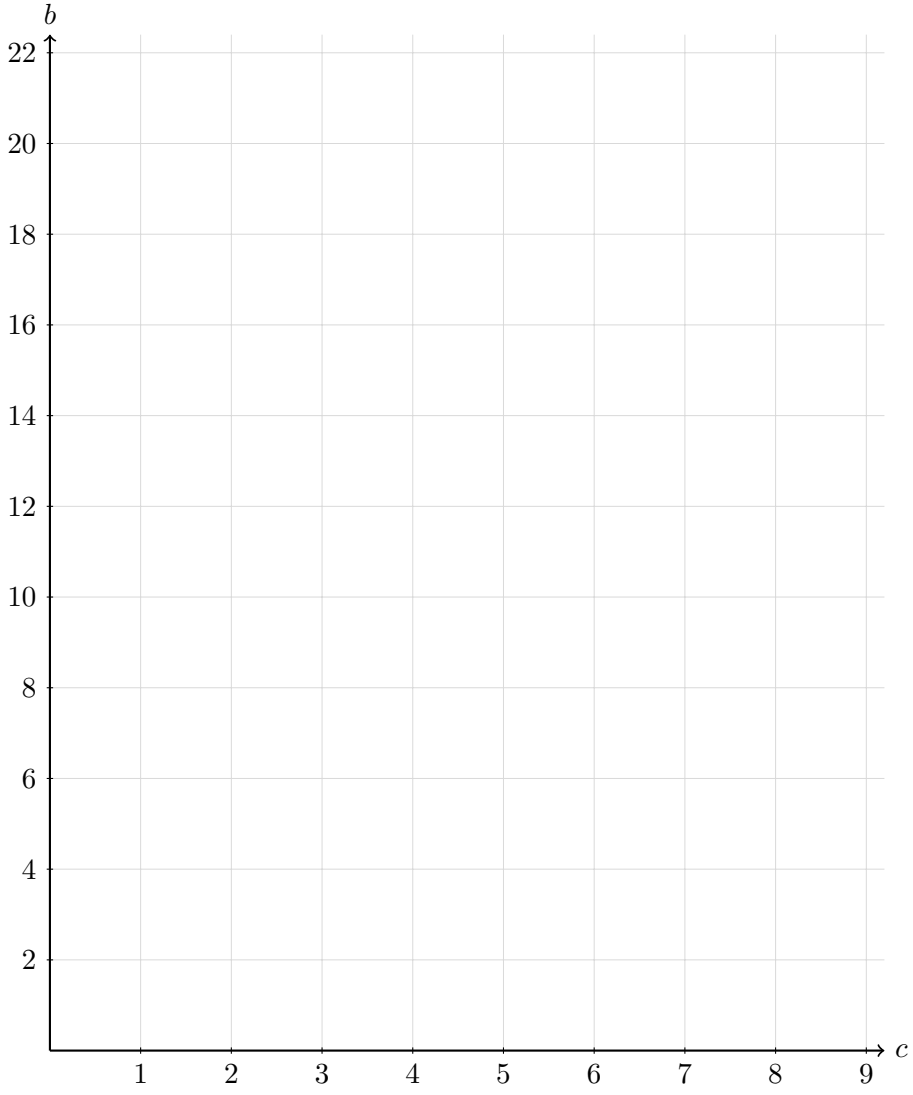
(e) Does the substitution effect cause Jeff to adopt more or less of each animal? By how much?

(f) After the price change and with his original income, how many of each animal does Jeff adopt?

(g) Does the income effect cause Jeff to adopt more or less of each animal? By how much?

(h) What is the total effect of the price decrease on the adoption of cats and bunnies?

(i) Graph all 3 of budget lines and indicate the 3 optimal bundles under each. Label the substitution and income effect for each animal.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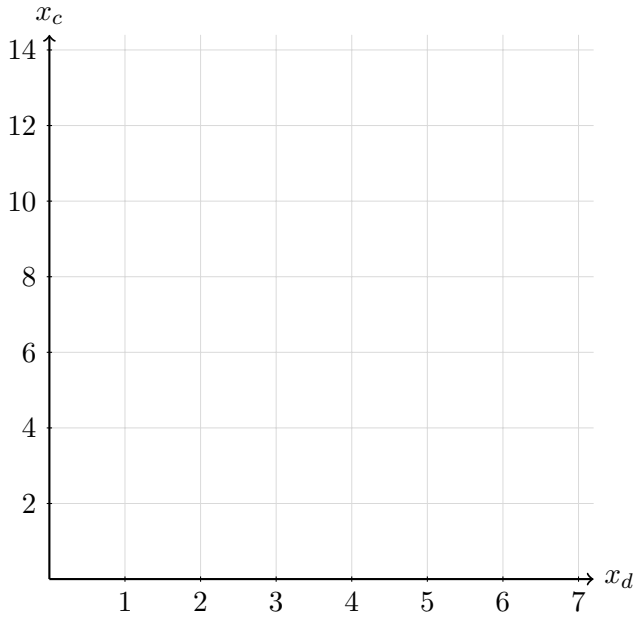
Problem 5. Stacy is headed to a chocolate fair with her endowment of 5 bars of dark chocolate (x_d) and 3 bars of caramel filled chocolate (x_c).

(a) Write down her budget line such that p_d and p_c represent the respective prices of the chocolate bars.

(b) Write down her budget line if the price of dark chocolate is \$5 and caramel filled is \$2.50. Graph it below and label her endowment point Ω.



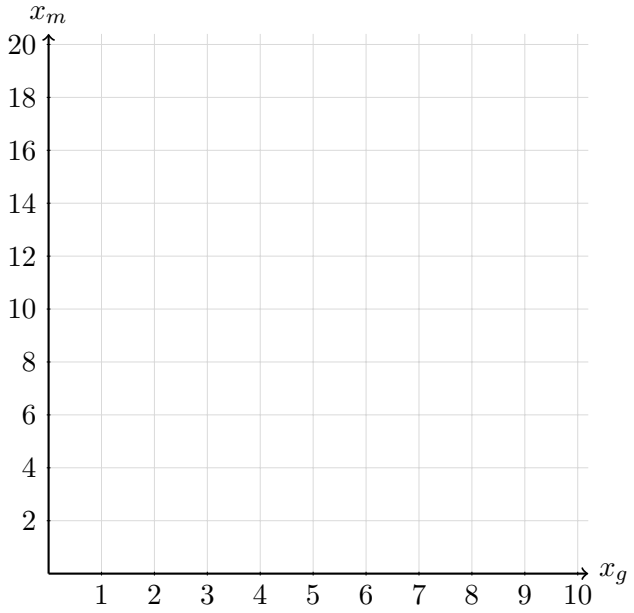
(c) Stacy left the chocolate fair with 3 dark chocolate bars and 7 caramel filled bars. What are Stacy's gross and net demands of each bar? Label her optimal bundle X^* on the graph.

(d) Is Stacy a net buyer or net seller of dark chocolate? What about for caramel filled?

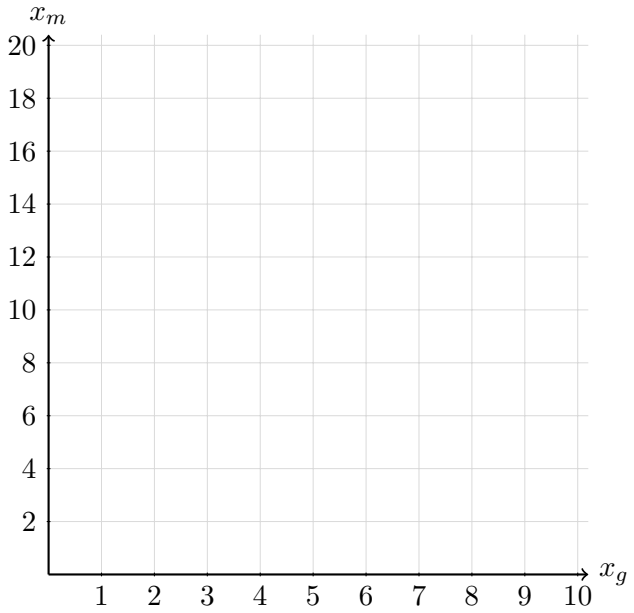
(e) If next week, the price of dark chocolate decreased to \$2.50 and Stacy instead leaves the fair with 6 dark chocolate bars and 2 caramel filled bars, is she worse off, better off, or is the effect of the price decrease ambiguous? Explain.

Problem 6. Steven likes to play golf (x_g) and go to the movies (x_m). He has \$200 to spend on the two goods with a round of golf costing \$40 and a movie costing \$10.

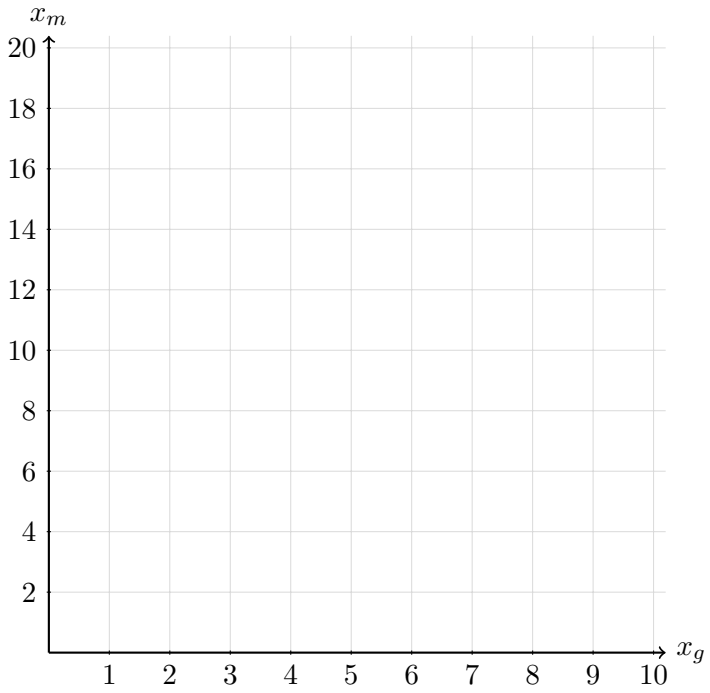
(a) What is Steven's budget constraint? Draw his budget line below and label it BL_1.



(b) Steven's wife has decided he has been playing too much golf recently and told him he cannot play more than 3 rounds. Draw his new rationed budget line below and label it BL_2.



(c) Suppose his wife no longer limits his golf playing. Instead, she makes him pay her \$20 for each round of golf he plays after the second round. Draw his new kinked budget line below and label it BL_3.



Problem 1. Will has the following utility function: $u(x_1, x_2) = \min\{x_1, 2x_2\}$

(a) Graph three of his indifference curves such that his utility equals 2, 5 and 10. Label them IC_1, IC_2, and IC_3 respectively.

$u(x_1, x_2) = 2$
$\min\{x_1, 2x_2\} = 2$
Kinked point is where $x = 2$ and $2y = 2 \implies (2, 1)$
Some other bundles that give a utility of 2 are: $(2, 2), (2, 6), (4, 1), (6, 1)$

$u(x_1, x_2) = 5$
$\min\{x_1, 2x_2\} = 5$
Kinked point is where $x = 5$ and $2y = 5 \implies (5, 2.5)$
Some other bundles that give a utility of 5 are: $(5, 5), (5, 9), (6, 2.5), (10, 2.5)$

$u(x_1, x_2) = 10$
$\min\{x_1, 2x_2\} = 10$
Kinked point is where $x = 10$ and $2y = 10 \implies (10, 5)$
Some other bundles that give a utility of 10 are: $(10, 6), (10, 10), (12, 5), (15, 5)$

(b) If the price of x_1 is \$2, x_2 is \$4, and Will has an income of \$20 what is his budget line? Graph it below and label it BL_1. What is Will's optimal bundle?

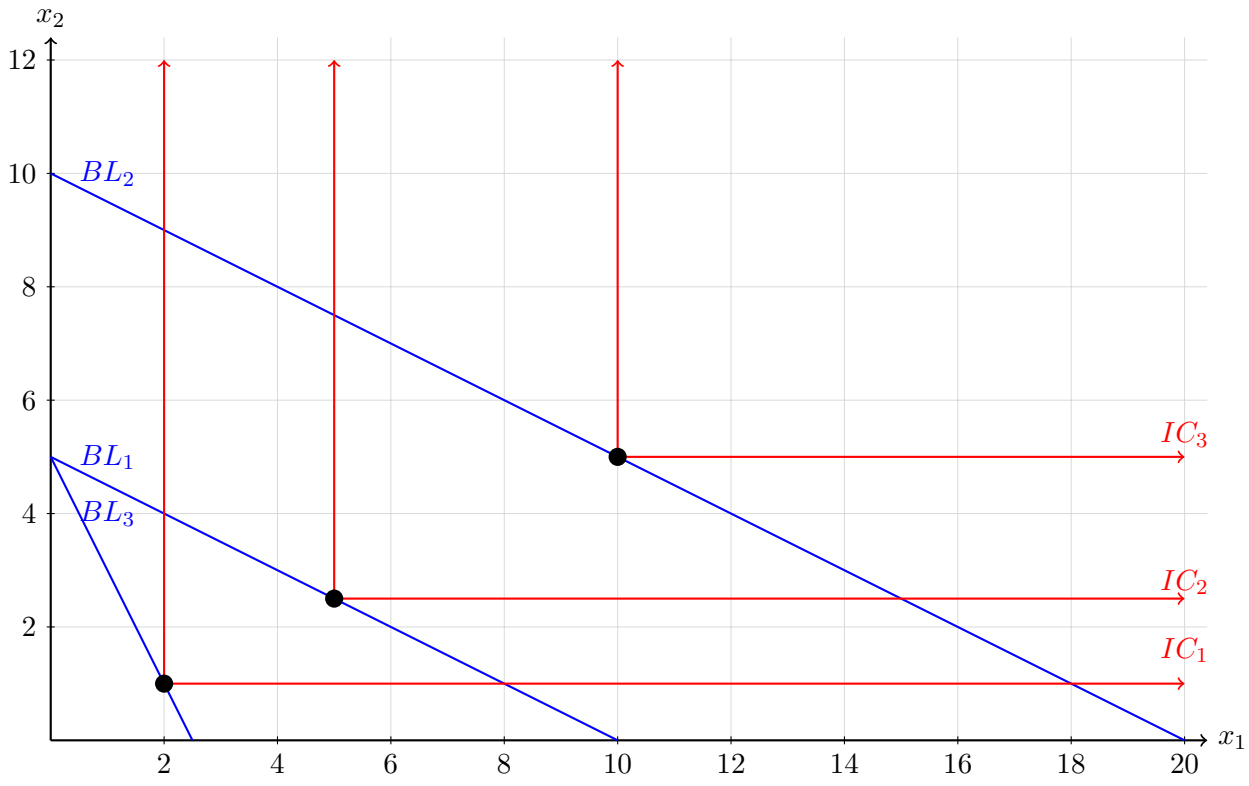
$BL_1 : 2x_1 + 4x_2 = 20$
Intercepts: $(0, 5)$ and $(10, 0)$
Optimal bundle: $(5, 2.5)$

(c) If the price of x_1 is \$2, x_2 is \$4, and Will has an income of \$40 what is his budget line? Graph it below and label it BL_2. What is Will's optimal bundle?

$BL_2 : 2x_1 + 4x_2 = 40$
Intercepts: $(0, 10)$ and $(20, 0)$
Optimal bundle: $(10, 5)$

(d) If the price of x_1 is \$8, x_2 is \$4, and Will has an income of \$20 what is his budget line? Graph it below and label it BL_3. What is Will's optimal bundle?

$BL_3 : 8x_1 + 4x_2 = 20$
Intercepts: $(0, 5)$ and $(2.5, 0)$
Optimal bundle: $(2, 1)$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Problem 2. Jon likes to spend his time ice sculpting and theorizing. His preferences over these two activities are represented by the utility function $u(x_t, x_i) = 2x_i + 50x_t - 2x_t^2$. His demand for theorizing is represented by the function $x_t(p_t, p_i) = 12.5 - \frac{p_t}{2p_i}$

(a) What is Jon's inverse demand of theorizing?

$x_t(p_t, p_i) = 12.5 - \frac{p_t}{2p_i}$ (Demand function)
$x_t = 12.5 - \frac{p_t}{2p_i}$ (Replace with quantity demanded)
$\frac{p_t}{2p_i} = 12.5 - x_t$ (Solve for p_t)
$p_t = 2p_i(12.5 - x_t)$ (Simplify)
$p_t(x_t, p_i) = 25p_i - 2p_i x_t$ (Distribute and rewrite as a function)

(b) If Jon has 200 hours of free time to spend and each ice sculpting takes 2 hours while theorizing takes 4 hours, what is his budget line? What is his optimal bundle?

$BL : 2x_i + 4x_t = 200$

$x_t(p_t, p_i) = 12.5 - \frac{p_t}{2p_i}$ (Demand function)
$x_t(4, 2) = 12.5 - \frac{4}{2(2)}$ (Plug in prices)
$x_t^* = 11.5$ (Solve)

$BL : 2x_i + 4(11.5) = 200$ (Substitute in the optimal amount of x_t into the budget line)
$2x_i = 154$ (Solve)
$x_i^* = 77$

Optimal bundle: $(11.5, 77)$

(c) Suppose instead that it takes Jon 2 hours to theorize, what is his new optimal bundle?

$BL : 2x_i + 2x_t = 200$

$x_t(p_t, p_i) = 12.5 - \frac{p_t}{2p_i}$ (Demand function)
$x_t(2, 2) = 12.5 - \frac{2}{2(2)}$ (Plug in prices)
$x_t^* = 12$ (Solve)

$BL : 2x_i + 2(12) = 200$ Substitute in the optimal amount of x_t into the budget line)
$2x_i = 176$ (Solve)
$x_i^* = 88$

Optimal bundle: $(12, 88)$

Problem 3. Raoul likes to consume fancy Italian espresso and coffee. His utility function over the two goods is $u(e, c) = 6e + 3c$. He has a budget of \$60 to spend on the two goods where the price of espresso is \$10 and coffee is also \$10.

(a) Write down and graph his budget line below. Label it BL_1

$BL : 10e + 10c = 60$
Intercepts: (0,6) and (6,0)

(b) Graph 3 of his indifference curves such that they represent a utility of 12, 24, and 36. Label them IC_1, IC_2, and IC_3 respectively.

$u(e, c) = 12$
$6e + 3c = 12$
Intercepts: (0,4) and (2,0)

$u(e, c) = 24$
$6e + 3c = 24$
Intercepts: (0,8) and (4,0)

$u(e, c) = 36$
$6e + 3c = 36$
Intercepts: (0,12) and (6,0)

(c) What kind of preference do these indifference curves represent?

Perfect Substitutes

(d) What is Raoul's optimal bundle?

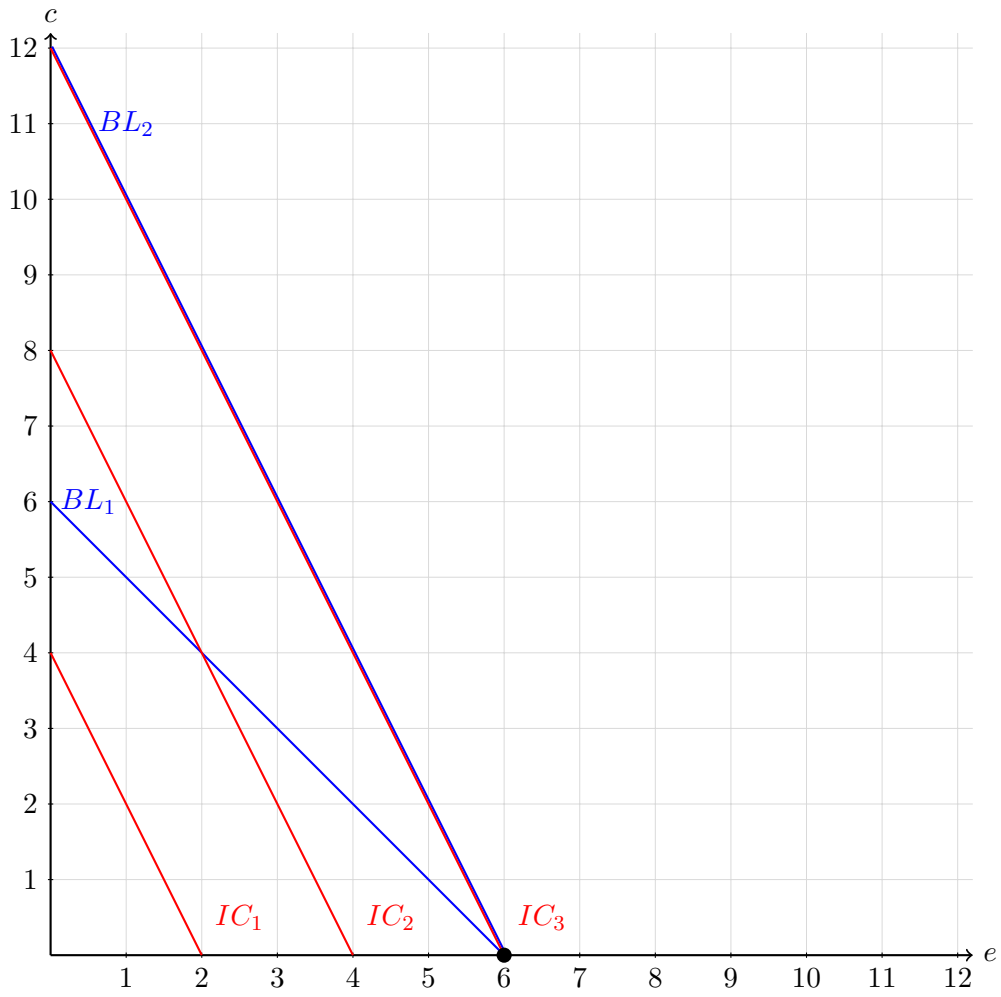
(6,0)

(e) Suppose the government issues a subsidy of \$5 for each bag of coffee beans bought from Italy. What is Raoul's new budget line. Graph it below and label it BL_2.

$BL_2 : (10e + (10 - 5)c = 60$
$BL_2 : 10e + 5c = 60$
Intercepts: (0,12) and (6,0)

(f) What is his new optimal bundle with the subsidy?

Any bundle on BL_2 can be an optimal bundle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Problem 4. Jeff is a big animal lover. Specifically, he likes to adopt cats (c) and bunnies (b). His preference over the two pets is represent by the utility function of $u(c, b) = 3c^2 + 2b$. Let m be his income, p_c the adoption price of cats, and p_b the adoption price of bunnies.

(a) If the current adoption prices are \$60 for cats and \$10 for bunnies and Jeff has an income of \$220 what is his budget line?

$BL : 60c + 10b = 220$

(b) If Jeff's demand function for cats is $c(p_c, p_b) = \frac{p_c}{3p_b}$, how many of each animal will he adopt?

$c(60, 10) = \frac{60}{3(10)}$ (Plug prices into the demand function)
$c^* = 2$ (Solve)

$BL : 60(2) + 10b = 220$ (Plug c^* into the budget line)
$10b = 100$ (Simplify)
$b^* = 10$ (Solve)

Optimal bundle: (2,10)

(c) If the adoption price of cats fell to \$40, what would Jeff's income need to be for him to still be able to afford his original optimal bundle? Denote this new income m'

$\Delta m = \Delta p_c(c)$
$\Delta m = (40 - 60)(2)$
$\Delta m = -40$
$m' = 220 - 40$
$m' = 180$

(d) Under the modified income and changed price, how many of each animal would Jeff adopt?

$c(40, 10) = \frac{40}{30}$ (Plug prices into the demand function)
$c^* = \frac{4}{3}$ (Solve)

$BL : 40c + 10b = 180$
$BL : 40(\frac{4}{3}) + 10b = 180$ (Plug c^* into the budget line)
$\frac{160}{3} + 10b = 180$ (Simplify)
$b^* = \frac{38}{3}$ (Solve)

Optimal Bundle:$(\frac{4}{3}, \frac{38}{3})$

(e) Does the substitution effect cause Jeff to adopt more or less of each animal? By how much?

The substitution effect causes Jeff to adopt $\frac{2}{3}$ fewer cats and $\frac{8}{3}$ more bunnies.

(f) After the price change and with his original income, how many of each animal does Jeff adopt?

$c(40, 10) = \frac{40}{30}$ (Plug prices into the demand function)
$c^* = \frac{4}{3}$ (Solve)

$BL: 40c + 10b = 220$
$BL: 40(\frac{4}{3}) + 10b = 220$ (Plug c^* into the budget line)
$\frac{160}{3} + 10b = 220$ (Simplify)
$b^* = \frac{50}{3}$ (Solve)

Optimal Bundle:$(\frac{4}{3}, \frac{50}{3})$

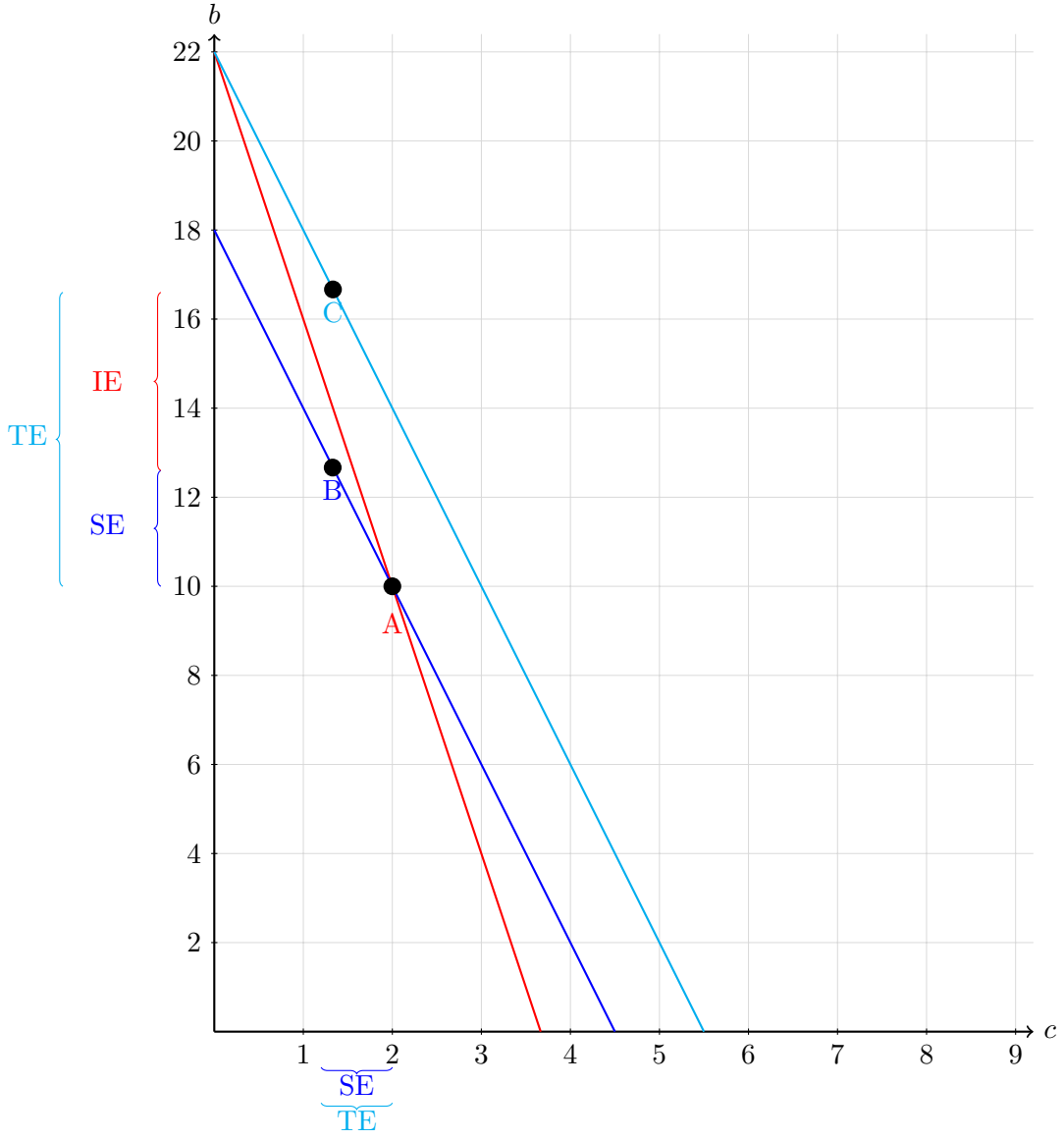
(g) Does the income effect cause Jeff to adopt more or less of each animal? By how much?

The income effect doesn't causes Jeff to adopt any more or less cats, but causes him to adopt 4 more bunnies.

(h) What is the total effect of the price decrease on the adoption of cats and bunnies?

The total effect is $\frac{2}{3}$ less cats and $\frac{20}{3}$ more bunnies

(i) Graph all 3 of budget lines and indicate the 3 optimal bundles under each. Label the substitution and income effect for each animal.



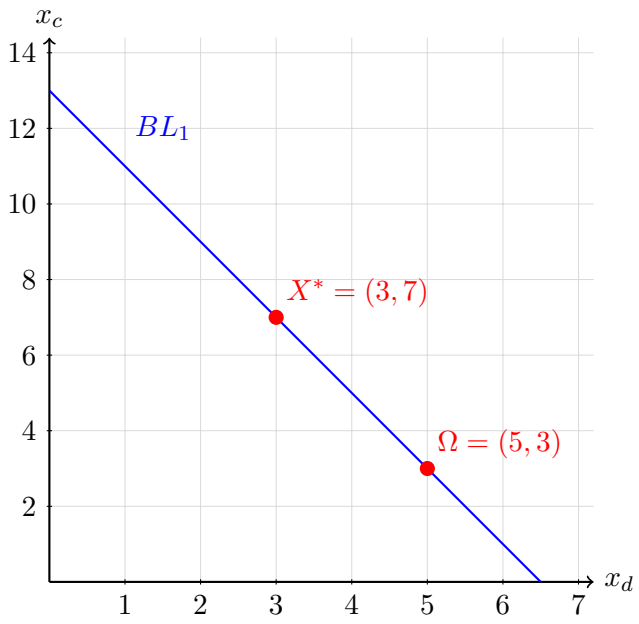
Problem 5. Stacy is headed to a chocolate fair with her endowment of 5 bars of dark chocolate (x_d) and 3 bars of caramel filled chocolate (x_c).

(a) Write down her budget line such that p_d and p_c represent the respective prices of the chocolate bars.

$BL : p_d x_d + p_c x_c = p_d(5) + p_c(3)$

(b) Write down her budget line if the price of dark chocolate is \$5 and caramel filled is \$2.50. Graph it below and label her endowment point Ω.

$BL : 5x_d + 2.50x_c = 5(5) + 2.50(3)$
$BL : 5x_d + 2.50x_c = 32.50$
Intercepts: (0,13) and (6.5,0)



(c) Stacy left the chocolate fair with 3 dark chocolate bars and 7 caramel filled bars. What are Stacy's gross and net demands of each bar? Label her optimal bundle X^* on the graph.
Gross Demand: $(3, 7)$
Net Demand: $(3 - 5, 7 - 3) = (-2, 4)$

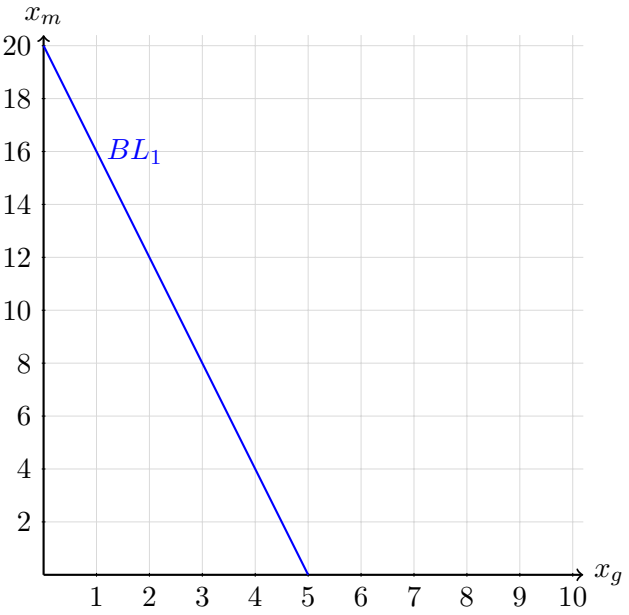
(d) Is Stacy a net buyer or net seller of dark chocolate? What about for caramel filled?
She is a net seller of dark chocolate and net buyer of caramel filled

(e) If next week, the price of dark chocolate decreased to \$2.50 and Stacy instead leaves the fair with 6 dark chocolate bars and 2 caramel filled bars, is she worse off, better off, or is the effect of the price decrease ambiguous? Explain.
New Net Demand: $(6 - 5, 2 - 3) = (1, -1)$
She is now a net buyer of dark chocolate and net seller of caramel filled. Since she was originally a net seller of dark chocolate and net buyer of caramel filled, her behavior toward both goods switched. Therefore, the effect of the price decrease is ambiguous.

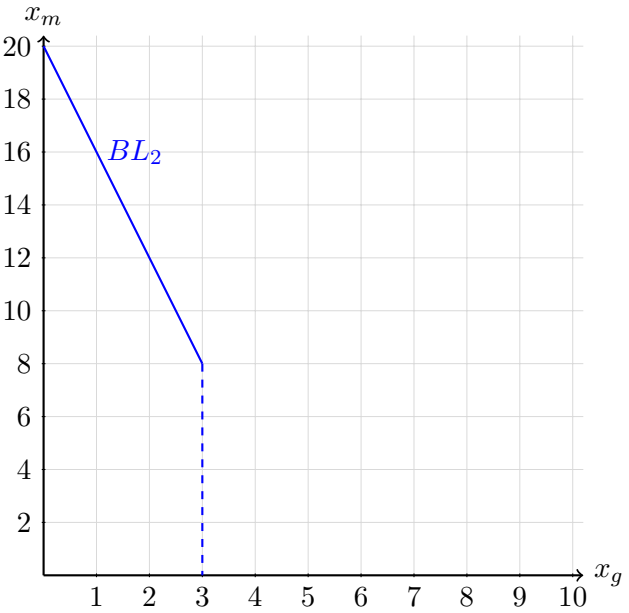
Problem 6. Steven likes to play golf (x_g) and go to the movies (x_m). He has \$200 to spend on the two goods with a round of golf costing \$40 and a movie costing \$10.

(a) What is Steven's budget constraint? Draw his budget line below and label it BL_1.

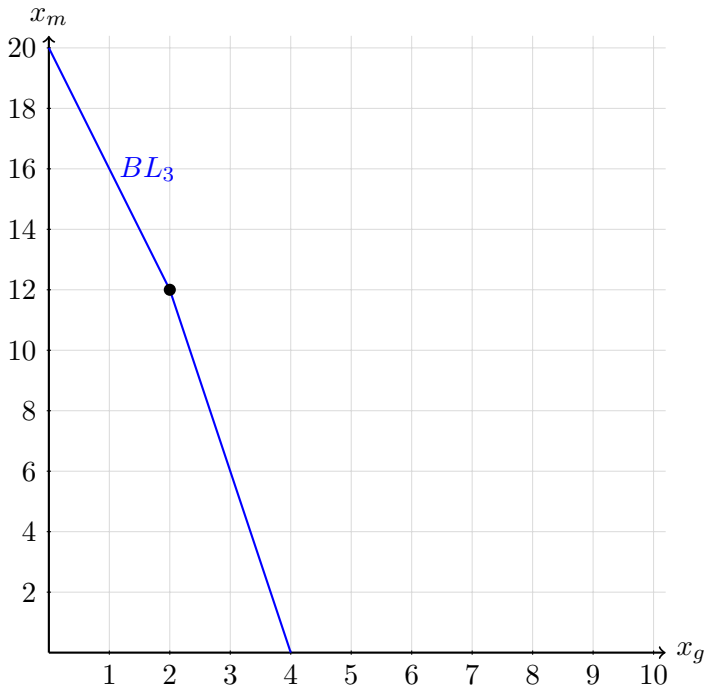
$BC : 40x_g + 10x_m \leq 200$
Intercepts: (0,20) and (5,0)



(b) Steven's wife has decided he has been playing too much golf recently and told him he cannot play more than 3 rounds. Draw his new rationed budget line below and label it BL_2.



(c) Suppose his wife no longer limits his golf playing. Instead, she makes him pay her \$20 for each round of golf he plays after the second round. Draw his new kinked budget line below and label it BL_3.



If Steven spends all his money on movies, he can buy:
$10x_m = 200$
$x_m = 20$
$(0, 20)$

If Steven buys 2 rounds of golf (the amount allowed before the price increase), then the amount of movies he can buy is:
$BL : 40(2) + 10x_m = 200$
$10x_m = 120$
$x_m = 12$
$(2, 12)$

If Steven spends all his money on rounds of golf, he can buy:
$40(2) = 80$ (Amount spent on rounds of golf before the price increase)
$60x_g = 120$(Amount spent on rounds of golf after the price increase)
$x_g = 2$
$x_g = 2 + 2 = 4$ (Total amount of rounds bought)
(4,0)